Journal of Applied Statistics, 2013
Vol. 40, No. 11, 2317–2331, http://dx.doi.org/10.1080/02664763.2013.810194

Taylor & Francis
Taylor & Francis Group

Temporal dynamics of inter-limb coordination in ice climbing revealed through change-point analysis of the geodesic mean of circular data

Ludovic Seifert[a], Jean-François Coeurjolly[b]*, Romain Hérault[c], Léo Wattebled[a] and Keith Davids[d]

[a]*Faculty of Sports Sciences, Centre d'Etude des Transformations des Activités Physiques et Sportives (CETAPS) – EA 3832, University of Rouen, Mont-Saint-Aignan, France;* [b]*Laboratory Jean Kuntzmann – UMR CNRS 5224, Grenoble University, Grenoble, France;* [c]*Laboratoire d'Informatique, de Traitement de l'Information et des Systèmes (LITIS) – EA 4108, National Institute of Applied Sciences (INSA), Rouen, France;* [d]*School of Human Movement Studies, Queensland University of Technology, Brisbane, Australia*

(*Received 9 July 2012; accepted 28 May 2013*)

This study examined the temporal dynamics of the inter-limb angles of skilled and less skilled ice climbers to determine how they explored ice fall properties to adapt their coordination patterns during performance. We observed two circular time series corresponding to the upper- and lower-limbs of seven expert and eight inexperienced ice climbers. We analyzed these data through a multiple change-point analysis of the geodesic (or Fréchet) mean on the circle. Guided by the nature of the geodesic mean obtained by an optimization procedure, we extended the filtered derivative method, known to be computationally very cheap and fast, to circular data. Local estimation of the variability was assessed through the number of change-points computed via the filtered derivatives with *p*-value method for the time series and integrated squared error (ISE). Results of this change-point analysis did not reveal significant differences of the number of change-points between groups but indicated higher ISE that supported the existence of plateaux for beginners. These results emphasized higher local variability of limb angles for experts than for beginners suggesting greater dependence on the properties of the performance environment and adaptive behaviors in the former. Conversely, the lower local variance of limb angles assessed in beginners may reflect their independence of the environmental constraints, as they focused mainly on controlling body equilibrium.

Keywords: variability analysis; inter-limb coordination; climbing; change-point analysis; geodesic mean; circular data

*Corresponding author. Email: jean-francois.coeurjolly@upmf-grenoble.fr

1. Context of the application and main objective

Traditionally, it has been argued that the acquisition of movement expertise is characterized by a linear progression towards invariance in motor output performance. From this perspective, movement pattern variability is an errorful by-product of noise in the central nervous system that should be minimized or eliminated with practice. This view is compounded by the types of motor tasks used to study movement performance in experiments, emphasizing deviations from an ideal performance template, characteristic of expert behavior [32,34]. However, from an ecological dynamics perspective, observed variability in motor output in both novice and expert individuals in performance domains like sport may not necessarily be a reflection of system error or noise. Expertise in sport results from the adaptation of behaviors to interacting constraints, individually perceived and encountered. Indeed, the intertwined relationship between perceptions and actions constrains the direction, and restrains the range of movement possibilities available for each individual performer. With this emphasis on perception and action to constrain behaviors, the role of movement pattern stability and functional intra-individual performance variability is paramount. Beek *et al.* [5] suggested that the nature of relationship and the coupling of perception and action is not the same for non-experts and experts, since the expert is more capable of exploiting information about task-related constraints in order to organize their behaviors.

In traditional research, movement expertise has been captured statistically through calculations of the magnitude of variance measures like the standard deviation of the mean distribution and the coefficient of variation [17,28]. These statistical indicators attempt to characterize the data distribution and the amount of noise in a single measurement pertaining to performance. However, such statistical measurements only indicate the magnitude of system variability (i.e. the amplitude and the spatial distribution of performance outcomes over trials), but not the dynamical structure of the data series [28]. Recent studies have explored the structure of variability for performance outcomes through identifying the learning dynamics, showing multiple time scales of variability (such as exponential, power law and S-shaped performance curves) [24,27,29,30]. According to the dynamical systems' approach, these multiple time scales emphasize the discontinuities that typify learning, based on the assumptions that learning is constructed from spontaneous manifestations of motor coordination and that often it is necessary to destabilize an established motor pattern in order to provoke the attainment of expert coordination [37]. For instance, it has been hypothesized [29] that when a system is close to its stable state, it will change at a constant time scale (i.e. an exponential function), while multiple time scales (i.e. a power law) are expected to be observed when movement coordination goes through transition. The emphasis in our study was on discovering statistical measures which capture the structure of movement pattern variability through observing the temporal dynamics of motor variability during an ice climbing ascent. In this paper we discuss new data analysis approaches which can demonstrate how studying expertise differences in sport can benefit from new statistical methodologies. Since movement patterns, during ice climbing, are predicated on ice fall properties (e.g. shape, steepness, temperature, thickness and ice density), an important question concerns how ice climbers of various skill levels exploit affordances (i.e. possibilities for action offered by a particular performance environment [18]) to organize their upper and lower movements over time. Our hypothesis is that ice fall properties contain affordances that induce variable motor coordination patterns in expert climbers, whereas learners use a basic and functionally stable motor organization to achieve their main goal of maintaining body equilibrium with respect to gravity.

The selected characteristics of different levels of expertise in ice climbing will be statistically described in Section 2. Our analysis has been based on the collection of angular time series data for eight beginners and seven experts. Such data fall into the general domain of circular statistics, for which there are general references [22,25,26]. The concepts of location and dispersion for circular statistics are really specific and, on a general manifold, we refer to other publications for a deep understanding of these concepts [9,10,20]. In this article, we have focused on intrinsic characteristics, i.e. on the geodesic mean and the geodesic variance. These parameters are the natural extensions of the standard Euclidean mean and the standard Euclidean variance when substituting the Euclidean distance by the geodesic distance on the circle; a distance which is referred to as the arc-length distance. We have recalled the main definitions of these concepts and apply this measure of dispersion in Section 2.4. As demonstrated in this section, based on computations of the sample geodesic variance, there was a clear distinction in behaviors between beginners and experts. However, this difference was mainly due to the fact that expert climbers explore a larger range of angle values than beginners. To have a better understanding of behaviors for both groups of climbers at different levels of expertise, we have turned to a local analysis of these data. In particular, we aimed to develop a change-point analysis of the geodesic mean for circular data.

The problem of change-point detection is certainly one of the most investigated issues by statisticians which has led to the development of a huge body of literature; for example [3,12,16] or, for more recent review, the article by Hušková and Meintanis [21]. It consists of detecting one or more time points where parameters of a process change. One of these methods, the filtered derivative, was initially introduced by Benveniste and Basseville [6], Basseville and Nikiforov [3], and Antoch and Hušková [1]. Generally speaking, this method consists of computing local estimations of the parameter of interest via a *filter* like a mean or a M-estimator, and in detecting changes in these local estimations through *derivation*. Extensions and theoretical studies have been considered by Bertrand *et al.* [7,8]. The statistical contribution of this article is to extend understanding of the filtered derivative with the p-value method in order to detect multiple change-points on the geodesic mean for circular data; this method is referred to as **`fdpv`** in the following sections. Regarding our data, and in particular the parameter of interest, it has raised the idea that the sample geodesic mean was actually an M-estimate obtained through an optimization procedure. Moreover, no sequential formula was available for such an estimate, i.e. the sample geodesic of the data set $(y_1, \ldots, y_n)$ could not be obtained using the sample geodesic mean of the data set $(y_1, \ldots, y_{n-1})$ and y_n. This convinced us to turn to a method that shows a very low complexity and a short running time. The main interest of the **`fdpv`** method that is used to detect abrupt changes in the standard Euclidean mean, variance or parameters of a simple linear regression is the fact that its time and complexity memory are both of order $\mathcal{O}(n)$.

The rest of the paper is organized as follows. We present in Section 2 the protocol from which the data have been obtained and the main characteristics of the climbers included in this study. We also specify the contribution of our data which is the observation of angular time series and introduce the concepts of geodesic mean and geodesic variance for circular data which constitute the parameters of interest of this paper. Section 3 turns to the core of the paper which is the extension to circular data of the filtered derivative method with p-values allowing us to propose an efficient change-point analysis method of the geodesic mean of a circular time series. Finally, we apply the developed methodology and discuss and interpret the results from a practical point of view in Section 4.

2. Description of the data and global variability analysis

2.1 *Participants*

Fifteen male ice climbers, divided into two groups, volunteered for this study. Seven expert climbers with mean age: 32.1 ($\sigma = 6.1$); mean height: 176.4 cm ($\sigma = 6.2$ cm); mean weight: 68.4 kg ($\sigma = 6.7$ kg); skill level in rock climbing: grade 7a+ to 7c on the French rating scale, which ranges from 1 to 9; mean number of years practicing rock climbing: 17.1 ($\sigma = 5.6$); skill level in ice fall climbing: grade 6–7 on the French rating scale, which goes from 1 to 7 [4]; mean number of years of practice in ice climbing: 10.4 ($\sigma = 4.7$); mean number of days of ice climbing per year: 20.6 ($\sigma = 9.3$). They were considered as skilled climbers since they were (i) mountain guides, certified by the International Federation of Mountain Guides Association (IFMGA) or/and (ii) instructors at the French National School of Skiing and Alpinism (ENSA). The eight beginners (mean age: 28.5 ($\sigma = 6.4$); mean height: 177.2 cm ($\sigma = 5.8$ cm); mean weight: 71.8 kg ($\sigma = 8.9$ kg)) were students in a Faculty of Sport Sciences at a local university, with 20 h of practice on an artificial climbing wall and were inexperienced at ice climbing.

2.2 *Protocol*

To impose a similar task constraint on both groups [28], a sub-maximal level of effort was imposed that corresponded to a 30 m ice fall climb at grade 5+ for expert climbers (which is a regular grade for them). The beginners climbed a 30 m ice fall at grade 4 (a common grade assigned to that skill level). Grade 5 + /6 signifies vertical climbing for most of the ice fall, while grade 4 involves alternation of steep sections around 80–85°, with ramps around 60–70°. For this protocol, the ice fall selected for the beginners was in three sections: 20 m at 85°, ramp of 5 m at 70°, then 5 m at 80°. Although a similar task constraint was imposed on the participants, these differences of grade between the two groups would represent different environmental constraints (i.e. in terms of steepness). Consequently, to enable a valid comparison between skilled climbers and beginners, the first 20 m part of the ice fall that corresponded to 85° of steepness for both groups was selected to analyze the motor behavior. Performance data were collected in two sessions during which the air temperature was, respectively, −8°C and −12°C. All climbers were equipped with the same crampons and ice tools and were instructed to climb at their normal pace. The protocol was approved by the University ethics committee and followed the declaration of Helsinki. Procedures were explained to the climbers, who then gave their informed consent to participate.

2.3 *Data collection*

A frontal camera (25 Hz), positioned 15 m behind the climber perpendicular to the ice fall, digitally recorded the first 20 m of the climb. A calibration frame delimited the recorded space of climbing performance and was composed of one vertical rope with marks every 2 m and two horizontal ropes (at 5 m and at 20 m) with marks every 1 m (total of 20 marks for calibration). Five key points (the head of left and right ice tools, and the extremity of left and right crampons) were digitized using Simi Motion Systems®(2004). Since climbing was self-paced, the time of ascent was not considered in assessing performance.

The nature and the number of ice tool and crampon actions completed during the ascent were counted, including (i) the ratio between definitive anchorage and repetitive ice tool swinging, and (ii) the ratio between definitive anchorage and repetitive crampon kicking.

Upper-limb coordination patterns were assessed by using the angle between the horizontal line and the displacement of the heads of the left and right hand ice tools. Lower-limb coordination patterns corresponded to the angle between the horizontal line and the displacement of the left and right crampons (Figure 1). These two signals were smoothed by a Butterworth low-pass filter (cut-off frequency 6.25 Hz) by Matlab 7.7®(1984–2008, The MathWorks, Inc.) as suggested by Winter [36] to address noise introduced by body marks digitizing from the video yet preserving movement information.

We highlighted eight angle modes, each of them are 45° span. When the angle was $0 \pm 22.5°$, the two limbs were horizontal, meaning that they were simultaneously flexed or simultaneously extended, corresponding to an in-phase mode of coordination. When the angle was $\pm 90 \pm 22.5°$, one limb was vertically located above the other limb, meaning that one was flexed, while the other was extended, corresponding to an anti-phase mode of coordination. Between these values, the limbs showed a diagonal angle so that coordination was considered in an intermediate mode. The

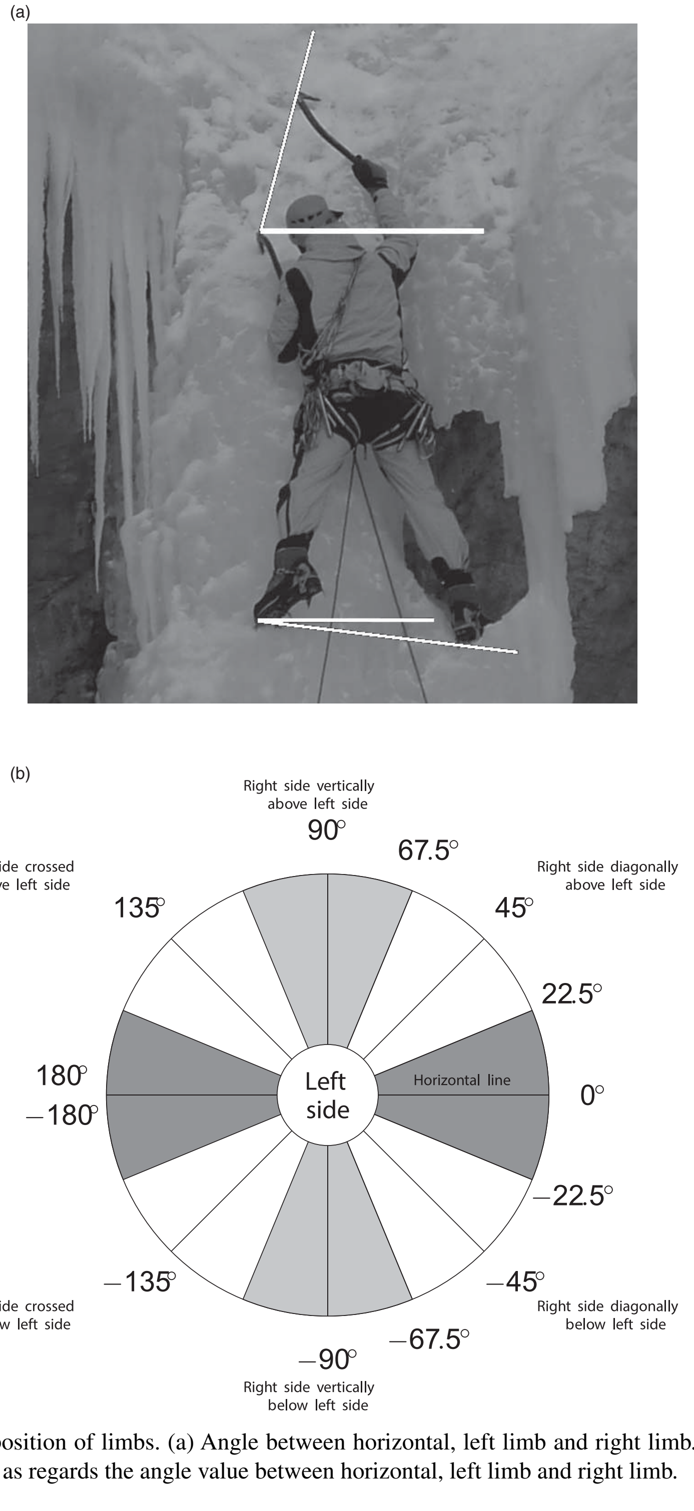

Figure 1. Angular position of limbs. (a) Angle between horizontal, left limb and right limb. (b) Modes of limbs' coordination as regards the angle value between horizontal, left limb and right limb.

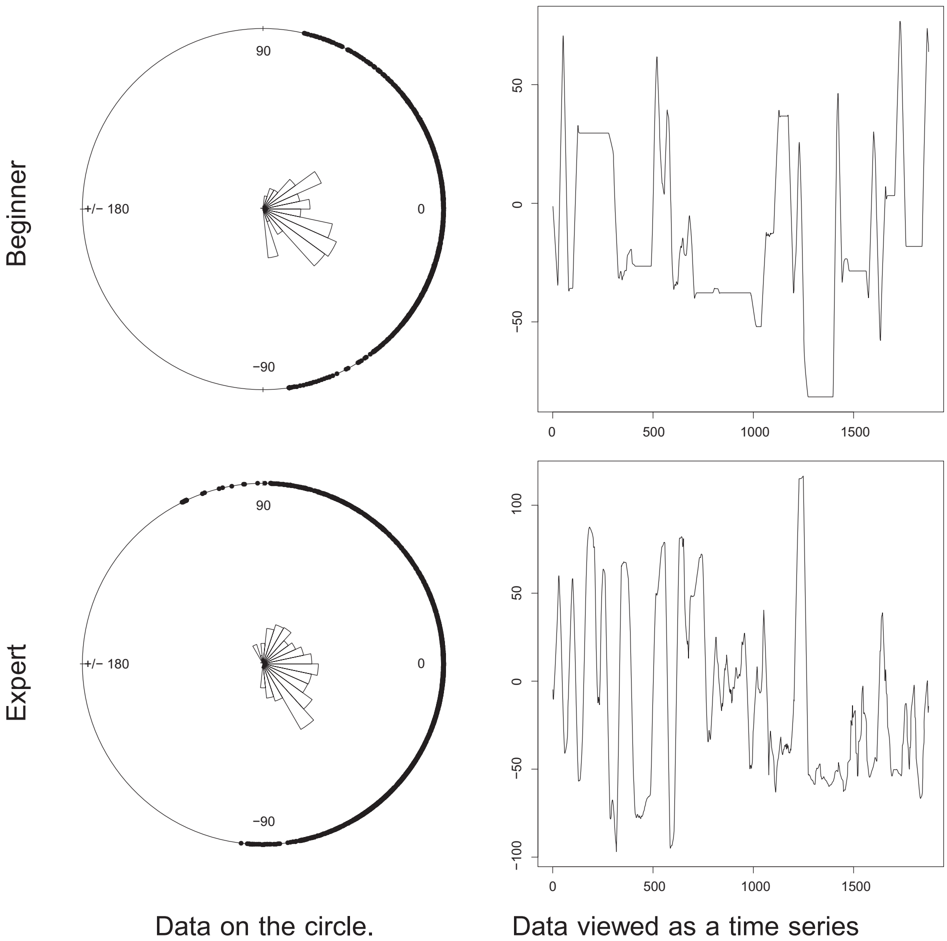

Figure 2. Angular data of the hand ice tools (i.e. time series ULC) for a beginner climber (top) and an expert one (bottom). The left plots represent the data on the circle. These plots and the associated rose diagram have been generated using the `R` package `circular`. The right plots represent the same data viewed as a time series (note that the vertical axis is not the same for the beginner and the expert climbers).

angle between the horizontal line and the left and right limbs was positive when the right limb was above the left limb and negative when the right limb was below the left limb, see Figure 1. To summarize, we collected two time series of angular data on performance for eight beginners and seven expert climbers: the first time series was related to the use of ice tools and represents upper-limb coordination, whereas the second set of data was related to the use of crampons, corresponding to lower-limb coordination. These time series, whose lengths are more or less 1874 data points (with a duration of 4 s between two points), will be, respectively, denoted by upper-limb coordination (ULC) and lower-limb coordination (LLC) in the following. Figure 2 illustrates these data for two different climbers and the next section aims to explore the statistical analysis of the variability characteristics in these data.

2.4 *Global variability analysis*

Due to the compactness of the circle (particular case of a compact manifold), the standard notions of mean and variance are not suited for circular data. Refs. [22,25,26] of the extant literature contain some detailed analyses of the concepts of location and dispersion for data on a circle (and for some of the references on a general manifold) (see [9,10,20]). Among these different concepts of location, we have focused in this article on the notions of a geodesic mean (or intrinsic mean) and geodesic variance (or intrinsic variability), that is on characteristics which are intrinsically defined via a distance on the manifold, here the circle S^1. In our opinion, these concepts seemed more natural than the classical extrinsic mean (obtained as the projection on the circle of the point with abscissa (resp. ordinate) equal to the mean of the cosine (resp. sine) of the angle) and extrinsic variance which are the concepts on which the general references [22,25,26] are based on.

For the specific manifold of the circle, denoted by S^1, the geodesic distance is the arc-length distance (expressed in degrees) which for two angles $(\alpha, \beta) \in [-180°, 180°)^2$ is expressed as

$$d_G(\alpha, \beta) = 180° - |180° - |\alpha - \beta||. \tag{1}$$

Now, given n observations $y_1, \dots, y_n$ of angles (expressed in radians units), the geodesic sample mean is defined by

$$\hat{\mu}_G = \operatorname*{Argmin}_{\mu \in S^1} \frac{1}{n} \sum_{i=1}^{n} d_G(y_i, \mu)^2 \tag{2}$$

and the geodesic sample variance by

$$\hat{\sigma}_G^2 = \frac{1}{n} \sum_{i=1}^{n} d_G(y_i, \hat{\mu}_G)^2, \tag{3}$$

which obviously satisfy $0 \leq \hat{\sigma}_G^2 \leq (180°)^2$. The sample geodesic mean and sample geodesic variance generalize in a very natural way the standard (Euclidean) sample mean and sample variance: the Euclidean distance is simply replaced by the geodesic distance. The notation μ_G and σ_G^2 stand for the theoretical geodesic mean and variance. The geodesic mean μ_G may not be unique, see [13,19,23] for a complete survey of this topic (on the circle). For example, the geodesic mean can be any point of the circle for an uniform distribution on $[-180°, 180°)$. Despite this, the sample geodesic mean is almost surely unique. Regarding the variances, the theoretical and the geodesic sample variances are necessarily unique. We applied this concept of dispersion for circular data to the time series ULC and LLC for all ice climbers.

Results are depicted in Figure 3 (top left). These findings make it clear that the global variance (i.e. the variance computed on the overall time series) was really linked to the intrinsic performance level of the climber. This clustering effect was actually mainly due to the large range of angles used by experts in contrast to beginners (see the top right plot of Figure 3). Indeed, if we had artificially rescaled the data such that the range of all the time series was $[-90°, 90°]$ (affine transformation applied to each angular time series), the geodesic variance would be less discriminant (see the right plot of Figure 3). To confirm these visual characteristics, we conducted a one-way analysis of variance (ANOVA; fixed factor: skill level). We obtained significantly higher geodesic variances of ULC ($F_{1,13} = 27.28, p = 0.0002$) and LLC ($F_{1,13} = 7.52, p = 0.0017$) for expert climbers than for beginners if we consider the raw data. However, based on the rescaled data, there was no clear evidence of significant different variances ($F_{1,13} = 2.72$, $p = 0.123$ for the ULC time series and $F_{1,13} = 1.81, p = 0.201$ for the LLC time series).

In conclusion, the global variability was almost linked to the global performance of the climber and did not really reflect the climber's style of performance. Figure 2 shows that expert climbers explored a larger range of angular positions of limbs than non-experts. Notably, according to the angular position classification of Figure 1, expert climbers exploited horizontal, diagonal, vertical and crossed angular positions while non-experts mostly used horizontal and diagonal angular positions.

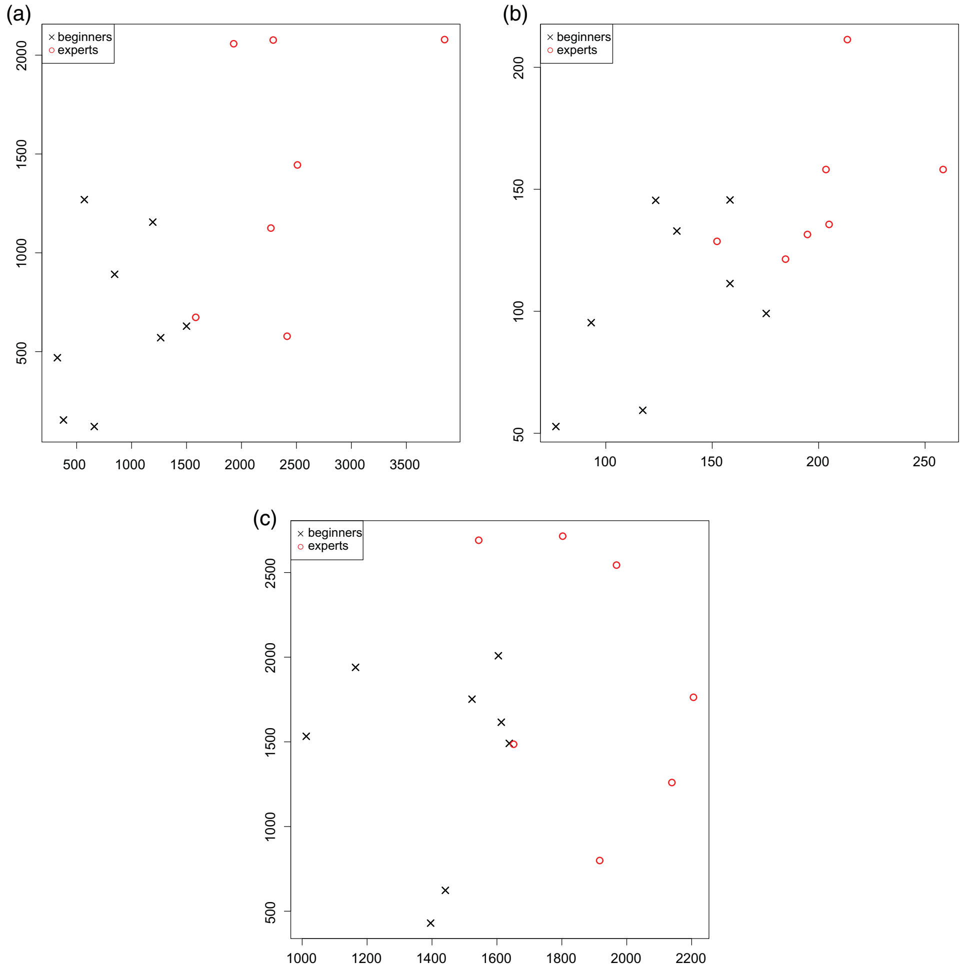

Figure 3. Upper-limb coordination versus lower-limb coordination. The abscissa (resp. the ordinates) correspond to the time series ULC (resp. LLC). (a) Geodesic variances of raw data. (b) Ranges of the time series (in °). (c) Geodesic variances on data artificially rescaled.

We demonstrate in the next section that a local estimation of the variability, based on a segmentation of the circular time series, provides more information on the behavioral differences in-between beginners and in-between expert climbers. And we highlight that the local estimation is almost independent of the global performance, i.e. independent of the support of the data.

3. Filtered derivative method for circular data

In this section, we extended a procedure based on filtered derivatives with a p-value to detect changes in the geodesic mean of circular data. Let $y_1, y_2, \ldots, y_n$ be the sample of a circular time series of length n. We decompose the methodology into two different steps: detection of potential change-points and then deletion of false alarms.

Step 1 Detection of potential change-points.

Roughly speaking, the filtered derivative method consists of computing local estimations of the parameter of interest and to detect changes on these local estimations. We define for some integer

$A \geq 1$ and for $k \in \{A+1, \ldots, n-A\}$ the statistic $D(k, A)$ by

$$D(k, A) := d_G(\hat{\mu}_G[(k-A+1) : k], \hat{\mu}_G[(k+1) : (k+A)]), \tag{4}$$

where $\hat{\mu}_G[i : j]$ (for $1 \leq i < j \leq n$) is the geodesic sample mean based on the observations $y_i, y_{i+1}, \ldots, y_{j-1}, y_j$. In [1] or [7], potential change-points are selected as times corresponding to the local maxima of the absolute value of the filtered derivative time series $D(\cdot, A)$, moreover, when this last quantity exceeds a given threshold. Then, in [8], the authors used a slightly different approach: a probability of a type I error is fixed at a level $p_1 \in (0, 1)$ and the corresponding threshold C_1 is given by

$$P\left(\underbrace{\max_{k \in [A, n-A]} D(k, A)}_{:=M(A)} > C_1 \mid H_0 \quad \text{is true}\right) = p_1, \tag{5}$$

where H_0 represents the null hypothesis, corresponding here to the absence of change-points. More specifically, H_0 corresponds to the situation where $y_1, \ldots, y_n$ are independent realizations of the same random variable Y. Bertrand *et al.* [8] considered the problem of detecting changes in the Euclidean mean, variance and parameters of the simple linear regression. For these problems, we managed to determine the asymptotic survival function of $M(A)$ under the assumption of independence of the observations when Y followed a distribution satisfying some moments conditions. The translation to circular data is not straightforward, and we address the estimation of parameters through two data-driven methods. Thus, regarding the nature of the parameter of interest (actually an M-estimate), we considered a parametric bootstrap approach and a non-parametric re-sampling method to estimate the survival function of $M(A)$:

(1) *Parametric bootstrap approach*. The asymptotic distribution in [8] obtained for the parameters (mean and variance) is obtained for a mild assumption on the distribution of the data. In this vein, we modeled the data by a specific parametric circular distribution, estimated the parameters using the maximum likelihood method and estimated the distribution of $M(A)$ using B replications of the circular distribution with estimated parameters. For this method, we have chosen the geodesic normal distribution on the circle, whose circular density rewrites $f(\theta) = k^{-1}(\gamma)e^{-(\gamma/2)(180/\pi)^2 d_G(\mu,\theta)^2}$ for some angle $\mu \in [-180°, 180°)$ and some real number $\gamma \geq 0$ and where $k(\gamma)$ is a normalizing constant given by $k(\gamma) = \sqrt{2\pi/\gamma}\, erf(\pi\sqrt{\gamma/2})$. The geodesic normal distribution was first introduced by Pennec [31] and defined for general Riemanian manifolds. In the particular case of the circle, Coeurjolly and Le Bihan [15] studied its statistical properties (moments, simulation, asymptotic properties of the maximum likelihood estimates, etc.). The choice of this circular distribution is guided by the fact that parameter μ corresponds to the geodesic theoretical mean of this distribution (our parameter of interest) and the MLE of μ is the sample geodesic mean.
(2) *Non-parametric re-sampling method*. Following advice in [2], the survival function is estimated by replications of the data obtained by permutations (B replications are used).

In our application (presented in Section 4), we set p_1 to 10% and used $B = 5000$ replications to estimate C_1 for each circular time series. We observed that for different values of A, both approaches lead to quite similar results (for each climber). Therefore, we only kept the permutation approach in the presentation of our empirical results.

Step 2. Deletion of false alarms.

Let $A \leq \tau_1 < \cdots < \tau_{\tilde{K}} \leq n - A$ be the $\tilde{K}$ change-points defined after Step 1. In this step, the signal is segmented into $\tilde{K}+1$ subsamples. The subsample k consists in the set $\{y_i | i \in [\tau_{k-1} +$

$1,\dots,\tau_k]\}$, called Y_k, which are the records of the time period $[\tau_{k-1}+1,\tau_k]$. As there is $\tilde{K}$ change-points, k can vary from 1 to $\tilde{K}+1$ setting $\tau_0=0$ and $\tau_{\tilde{K}+1}=n$.

To delete false alarms, Bertrand *et al.* [8] then proposed to test the parameter of interest (mean, variance) between two successive subsamples Y_k and Y_{k+1}, that is between $\{y_i|i\in[\tau_{k-1}+1,\dots,\tau_k]\}$ and $\{y_j|j\in[\tau_k+1,\dots,\tau_{k+1}]\}$.

In other words, $\tilde{K}$ statistical tests are formed and change points were kept if the corresponding p-value was lower than a fixed value p_2. In our setting, we modeled each signal on the period $[\tau_{k-1}+1,\tau_k]$ by independent observations of a circular random variable Y_k for $k=1,\dots,K+1$, we let $\tilde{\mu}_k$ denote the geodesic mean of Y_k and defined θ_k as $d_G(\tilde{\mu}_k,\tilde{\mu}_{k+1})$. Then, we considered the $\tilde{K}$ statistical tests

$$H_0:\theta_k=0 \quad \text{versus} \quad H_1:\theta_k\neq 0. \tag{6}$$

To be close to the nature of the data, we again proceeded with these different statistical tests using re-sampling methods ($B=5000$ permutation tests). For this second step, we followed the advice in [8] setting p_2 to the value 10^{-6}. With a slight alteration of notation, the final change-points are denoted by $\tau_1,\dots,\tau_{\check{K}}$ with $A\leq\tau_1<\cdots<\tau_{\check{K}}\leq n-A$ and $\check{K}\leq\tilde{K}$.

4. Results and discussion

4.1 *Numerical results*

The algorithm implemented in the R software and described in the previous section was applied to the $2\times 15=30$ circular time series. The window parameter has been set to $A=40$ for the 30 time series. We selected $A=40$ because the realization of one action of each limb (i.e. left arm swing, right arm swing, left foot kick and right foot kick) took in average 40 points (i.e. a duration of 10 s). Apart from this empirical choice, we would like to underline that the procedures have also been applied with window sizes from $A=25$ to $A=60$. The obtained results were quite similar to the findings presented later with the choice $A=40$, this stability can be explained by the fact that the Step 2 cancels many false discoveries.

We denote by $\hat{\mu}_{G,t}$ the piecewise constant function at time t estimated from the change-point analysis given by

$$\hat{\mu}_{G,t}=\sum_{k=1}^{\check{K}+1}\hat{\mu}_G[(\tau_{k-1}+1):\tau_k]\mathbf{1}(t\in[\tau_{k-1}+1,\tau_k]),$$

where, we set by convention, $\tau_0=0$ and $\tau_{\check{K}+1}=n$. In order to quantify the local variations of the data around $\hat{\mu}_G$, we propose to define the following criterion:

$$\text{ISE}=\sum_{t=1}^{n}d_G(y_t,\hat{\mu}_{G,t})^2=\sum_{k=1}^{\check{K}+1}(\tau_k-(\tau_{k-1}+1))\sum_{t=\tau_{k-1}}^{\tau_k}d_G(y_t,\hat{\mu}_G[(\tau_{k-1}+1):\tau_k])^2. \tag{7}$$

As a general comment, we noted that the second step of the **fdpv** method has allowed us to delete between 1 and 3 potential change-points proposed by the first step. The computational aspect was not negligible since due to the huge number of calculations of geodesic sample means and due to the re-sampling procedures, the **fdpv** method required about 1 h for one time series (of length $n=1874$).

Figure 4 presents an example of the segmentation method with two data sets and their related time series $D(t,A)$ allowing to detect changes. All the segmentations can be found in the supplementary material available online accompanying this paper. Table 1 aims at summing up the numerical results. The number of change-points (after the second step) and the integrated squared

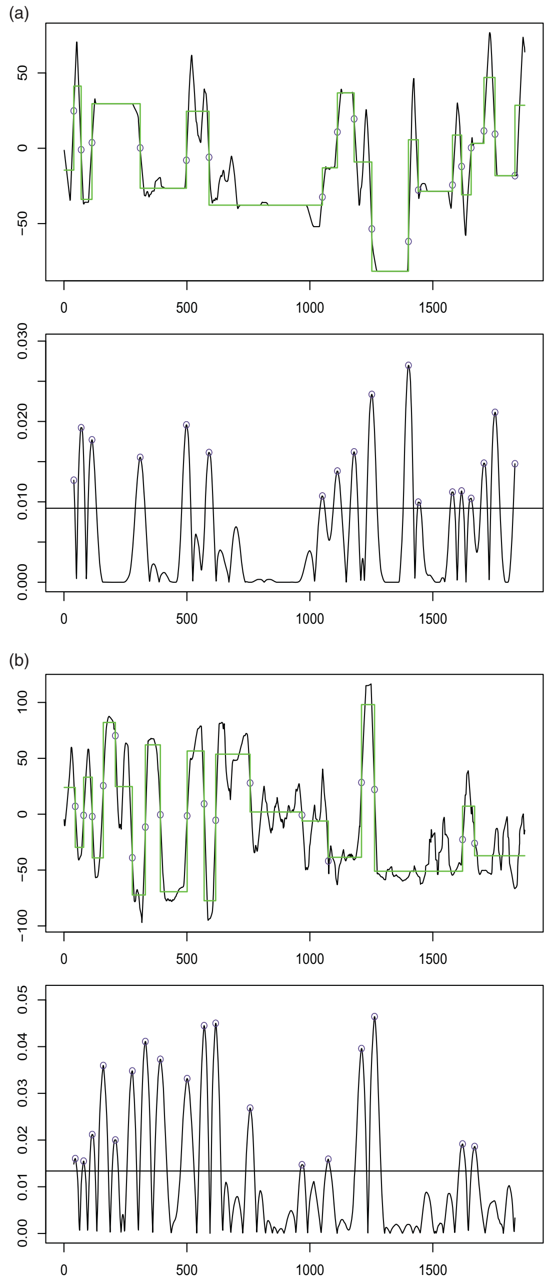

Figure 4. Examples of change-point analysis of the circular time series based on the filtered derivatives method with p-value. The data correspond to Figure 2. The two upper plots represent the selected change-points in blue circle and the resulting piecewise constant function (corresponding to the geodesic means computed on each segment). The bottom plots represent the related plot of the statistic $D(k, A)$ in terms of k. The horizontal line corresponds to the threshold C_1. For both examples, all change-points selected after the Step 1 were kept after the second step. The parameters of the method were $A = 40, p_1 = 10\%$ and $p_2 = 10^{-4}\%$.

Table 1. Summary results for the 15 ice climbers: number of change points $\check{K}$ computed via the filtered derivatives with p-value method for the time series ULC and LLC (first two columns) and integrated squared error denoted by ISE and defined by Equation (7).

	$\check{K}$		ISE	
	ULC	LLC	ULC	LLC
Beginner 1	18	13	65.4	59.4
2	9	12	27.7	3.0
3	9	12	24.2	5.9
4	20	16	21.3	28.8
5	18	15	65.8	69.0
6	15	20	61.5	31.7
7	24	16	106.6	30.9
8	23	14	79.7	61.0
Average	17	14.8	56.5	36.2
Expert 1	18	12	160.9	141.4
2	23	17	205.9	144.3
3	24	24	120.3	83.7
4	16	17	227.6	134.8
5	22	17	131.6	104.0
6	14	14	153.3	66.0
7	16	7	150.3	28.7
Average	19	15.4	164.3	97.5

error (ISE) criterion are presented. Whereas the number of change-points was not really different between a beginner and an expert climber, we highlight that the ISE was very discriminating. The low values of the ISE criteria were related to the existence of plateaux for beginners. This will be discussed in the next section.

Figure 3, presented in Section 2, shows in particular that the discrimination power of the global geodesic variance (i.e. computed on the overall data set) was essentially due to the larger range used by experts. After artificially rescaling the data, the global geodesic variance became much less discriminant. Figure 5 illustrates the counterpart of such analysis based on the ISE criterion. Simple statistics have been applied to the data in Table 1: a one-way ANOVA (fixed factor: skill level) showed significantly higher ISE of ULC ($F_{1,13} = 36.53, p < 0.0001$) and LLC ($F_{1,13} = 10.89, p = 0.006$) for expert climbers than for beginners. We highlight that our analysis is much less affected by a rescaling of the data. Indeed, we applied our general methodology to the artificially rescaled data (such that the range is $[-90°, 90°]$). We did not report the results but after undertaking the similar one-way ANOVA, we still obtained significantly higher ISE of ULC ($F_{1,13} = 19.83, p = 0.0006$) and LLC ($F_{1,13} = 9.57, p = 0.009$) for expert climbers than for beginners. There was clear evidence that the change-point analysis is less sensitive to the support of the time series which pertinently signifies in particular that locally the variability of angles for a beginner is much lower than for an expert.

As suggested previously, differences about ULC and LLC numbers could come from different number of action ratios between arm and foot; notably expert climbers often realized 1–3 foot kicks for 1 arm swing; while beginners realized 1–3 arm swings for 1 foot kick. However, we did not compute these data for this study as we did not examine arm to leg coupling.

4.2 *Interpretation and discussion*

Even if the global geodesic variance indicated significant differences between the two groups of climbers, these differences are linked to the range of angles used by climbers rather than to

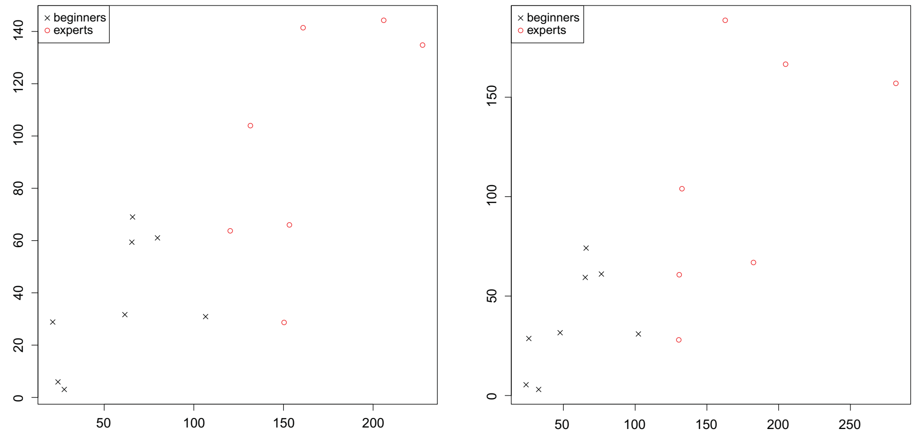

Figure 5. ISE defined by Equation (7) based on the change-point analysis of the circular time series for the 15 ice climbers. The left plot corresponds to the results based on raw data, also presented in the last two columns of Table 1. The right plot corresponds to the methodology based on data artificially rescaled in the interval $[-90°, 90°]$. The abscissa (resp. the ordinates) correspond to the time series ULC (resp. LLC).

the variability of the angles distribution. Therefore, the analysis of the local variance was more powerful to highlight significantly higher ISE for expert climbers than for non-experts, reflecting different behavioral adaptations to environmental constraints (i.e. ice fall properties). The longer duration spent without any movement of limbs statistically identified in the non-expert could have several causes:

(1) They spend more time to determine their climbing path and their next point of anchorage, suggesting their difficulty to perceive an affordance.
Qualitative analysis of video footage revealed that the non-experts swung their ice tool to create a hole in the ice fall whereas natural holes existed close to them. Conversely, expert climbers showed a greater dependence on the environment as they were able to exploit the ice fall properties (e.g. hole in the ice fall) to vary their limb angular positions and their limb movement patterns (e.g. swinging, kicking and hooking). Behavior variability corresponds to adaptive perception–action coupling to climb quickly, efficiently and safely. For instance, video footage showed that experts adopted vertical limb angular positions and sometimes crossed their limbs to hook existing hole in the ice fall and to use with their crampons the holes previously created with their ice tools.
(2) Non-experts spent more time to stabilize their body as they focused on keeping their body equilibrium under control (according to the findings of Bourdin *et al*. [11] in rock climbing), suggesting their relative independence of the environment. Body movements could be perceived as a potential cause of a fall; therefore beginners seemed to try and control body roll, yaw and pitch by freezing the motor system's degrees of freedom (as already observed in ski simulator tasks, [35]). Conversely, experts released the degrees of freedom to reach greater range of limb motion and length of vertical body displacement. Moreover, experts were able to exploit gravity (environmental constraint) by yaw and roll body motion leading the body to move like a pendulum or a door. The capacity of the expert to vary their movement patterns and limb angular positions revealed multi-stability of movement, that is a property of non-linear dynamical systems [14].
(3) Last, the non-experts needed a confident anchorage that was often synonymous with a deep anchorage. The high number of ice tool swinging and crampon kicking movements, and the

high ratio between swinging actions and definitive anchorages supported this impression. In particular, our results indicated that experts realized one ice tool swinging for one definitive anchorage, and one crampon kicking for one definitive anchorage. Conversely, for non-expert climbers, the ratio between definitive anchorages and swinging or kicking actions was 0.6 for ice tools and 0.2 for crampons. The non-expert climbers swung their ice tools two times and their crampons five times before a definitive anchorage. Therefore they spent a long time in a static body position leading to the onset of fatigue. This observation is in accordance with previous research in rock climbing [33] which relates to the 'three-holds-rule': if a rock climber uses a smaller number of holds he/she has to be quick enough to maintain equilibrium on the surface. Conversely, if the number of holds is equal to or greater than three, it is more likely that the rock climber will climb slowly, because his/her equilibrium is always under control [33].

5. Conclusion

Our study provided a valuable method to assess circular data in the sport performance domain, in particular the structure of variability through break-points in the upper-limbs and lower-limbs angle time series. Our results of this change-point analysis indicated higher levels of variability of limb angles for experts than for beginners suggesting greater dependence on the properties of the performance environment and adaptive behaviors in expert climbers. Conversely, the lower variance of limb angles assessed in beginners may reflect their independence of the environmental performance constraints, since they focused on controlling body equilibrium. Finally, by a structural analysis of variability, our method enabled the detection and understanding of the break-points causing plateaux and a lack of climbing fluency in order to improve the learning process in sport.

References

[1] J. Antoch and M. Hušková, *Procedures for the detection of multiple changes in series of independent observations*, Insur.: Math. Econ. 16 (1995), pp. 268–268.

[2] J. Antoch and M. Hušková, *Permutation tests in change point analysis*, Stat. Probab. Lett. 53 (2001), pp. 37–46.

[3] M. Basseville and I. Nikiforov, *Detection of Abrupt Changes: Theory and Application*, Prentice Hall Information and System Sciences Series, Prentice Hall Inc., Englewood Cliffs, NJ, 1993.

[4] P. Batoux and L. Seifert, *Ice Climbing and Dry Tooling: From Leman to Mont Blanc*, JM Editions, Chamonix, France, 2007.

[5] P. Beek, D.M. Jacobs, A. Daffertshofer, and R. Huys, *Expert performance in sport: Views from the joint perspectives of ecological psychology and dynamical systems theory*, in *Expert Performance in Sports: Advances in Research on Sport Expertise*, J.L. Starkes and K.A. Ericsson, eds., Human Kinetics Publishers, Champaign, IL, 2003, pp. 321–344.

[6] A. Benveniste and M. Basseville, *Detection of abrupt changes in signals and dynamical systems: Some statistical aspects,* in *Analysis and Optimization of Systems*, Lecture Notes in Control and Information Sciences, LNCIS-62, A. Bensoussan and J.L. Lions, eds., Springer-Verlag, Berlin, DE, 1984, pp. 145–155.

[7] P. Bertrand, *A local method for estimating change points: The 'hat-function'*, Stat.: J. Theor. Appl. Stat. 34 (2000), pp. 215–235.

[8] P. Bertrand, M. Fhima, and A. Guillin, *Off-line detection of multiple change points by the filtered derivative with p-value method*, Sequential Anal. 30 (2011), pp. 172–207.

[9] R. Bhattacharya and V. Patrangenaru, *Large sample theory of intrinsic and extrinsic sample means on manifolds: I*, Ann. Stat. 31 (2003), pp. 1–29.

[10] R. Bhattacharya and V. Patrangenaru, *Large sample theory of intrinsic and extrinsic sample means on manifolds: II*, Ann. Stat. 33 (2005), pp. 1225–1259.

[11] C. Bourdin, N. Teasdale, V. Nougier, C. Bard, and M. Fleury, *Postural constraints modify the organization of grasping movements*, Hum. Mov. Sci. 18 (1999), pp. 87–102.

[12] B. Brodsky and B. Darkhovsky, *Nonparametric Methods in Change-Point Problems*, vol. 243, Mathematics and Its Applications, Kluwer, Dordrecht, 1993.

[13] B. Charlier, *Necessary and sufficient condition for the existence of a fréchet mean on the circle*, Arxiv preprint (2011). Available at arXiv:1109.1986.

[14] J. Chow, K. Davidsb, R. Hristovskic, D. Araújod, and P. Passosd, *Nonlinear pedagogy: Learning design for self-organizing neurobiological systems*, New Ideas Psychol. 29 (2011), pp. 189–200.

[15] J.F. Coeurjolly and N. Le Bihan, *Geodesic normal distribution on the circle*, Metrika 75 (2012), pp. 977–995.

[16] M. Csörgö and L. Horváth, *Limit Theorems in Change-Point Analysis*, Wiley, Chichester, 1997.

[17] K. Davids, S. Bennett, and K. Newell, *Movement System Variability*, Human Kinetics Publishers, Champaign, IL, 2006.

[18] J. Gibson, *The Ecological Approach to Visual Perception*, Houghton-Mifflin, Boston, MA, 1979.

[19] T. Hotz and S. Huckemann, *Intrinsic means on the circle: Uniqueness, locus and asymptotics*, Arxiv preprint (2011). Available at arXiv:1108.2141.

[20] S. Huckemann, T. Hotz, and A. Munk, *Intrinsic shape analysis: Geodesic PCA for Riemannian manifolds modulo isometric lie group actions*, Stat. Sin. 20 (2010), pp. 1–58.

[21] M. Hušková and S. Meintanis, *Change-point analysis based on empirical characteristic functions of ranks*, Sequential Anal. 25 (2006), pp. 421–436.

[22] S. Jammalamadaka and A. Sengupta, *Topics in Circular Statistics*, World Scientific Pub. Co. Inc., Singapore, 2001.

[23] D. Kaziska and A. Srivastava, *The Karcher mean of a class of symmetric distributions on the circle*, Stat. Probab. Lett. 78 (2008), pp. 1314–1316.

[24] Y.T. Liu, G. Mayer-Kress, and K.M. Newell, *Qualitative and quantitative change in the dynamics of motor learning*, J. Exp. Psychol.: Hum. Percept. Perform. 32 (2006), pp. 380–393.

[25] K. Mardia, *Statistics of Directional Data*, Academic Press, London, 1972.

[26] K. Mardia and P. Jupp, *Directional Statistics*, Wiley, Chichester, 2000.

[27] P. Molenaar and K. Newell, *Individual Pathways of Change: Statistical Models for Analyzing Learning and Development*, American Psychological Association, Washington, DC, 2010.

[28] K. Newell, *Constraints on the development of coordination*, in *Motor Development in Children: Aspects of Coordination and Control*, M. Wade and H.T.A. Whiting, eds., vol. 34, Martinus Nijhoff, Dordrecht, 1986, pp. 341–360.

[29] K.M. Newell, Y.T. Liu, and G. Mayer-Kress, *Time scales in motor learning and development*, Psychol. Rev. 108 (2001), pp. 57–82.

[30] K.M. Newell, G. Mayer-Kress, S.L. Hong, and Y.T. Liu, *Adaptation and learning: Characteristic time scales of performance dynamics*, Hum. Mov. Sci. 28 (2009), pp. 655–687.

[31] X. Pennec, *Intrinsic statistics on Riemannian manifolds: Basic tools for geometric measurements*, J. Math. Imaging Vis. 25 (2006), pp. 127–154.

[32] R. Schmidt and T. Lee, *Motor Control and Learning: A Behavioral Emphasis*, 5th ed., Human Kinetics Publishers, Champaign, IL, 2011.

[33] F. Sibella, I. Frosio, F. Schena, and N.A. Borghese, *3d analysis of the body center of mass in rock climbing*, Hum. Mov. Sci. 26 (2007), pp. 841–852.

[34] J. Summers and J. Anson, *Current status of the motor program: Revisited*, Hum. Mov. Sci. 28 (2009), pp. 566–577.

[35] B. Vereijken, R.E.A. van Emmerika, H.T.A. Whitingb, and K.M. Newellc, *Freezing degrees of freedom in skill acquisition*, J. Motor Behav. 24 (1992), pp. 133–142.

[36] D. Winter, *Biomechanics of Human Movement*, Wiley, New York, 1979.

[37] P. Zanone and J. Kelso, *Evolution of behavioral attractors with learning: Nonequilibrium phase transitions*, J. Exp. Psychol.: Hum. Percept. Perform. 18 (1992), pp. 403–421.